PRIME MINISTER'S DEPARTMENT
DEPARTMENT OF STATISTICS MALAYSIA

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Non-linear Regression of Pumping Rate for Jenderam Hilir Heterogeneous Aquifer

Umi Haryantie Binti Haji Mohd Hisham
Supervisors: 1. Prof. Madya Dr. Arifah Binti Bahar
2. Dr Mohd Khairul Nizar Bin Shamsuddin

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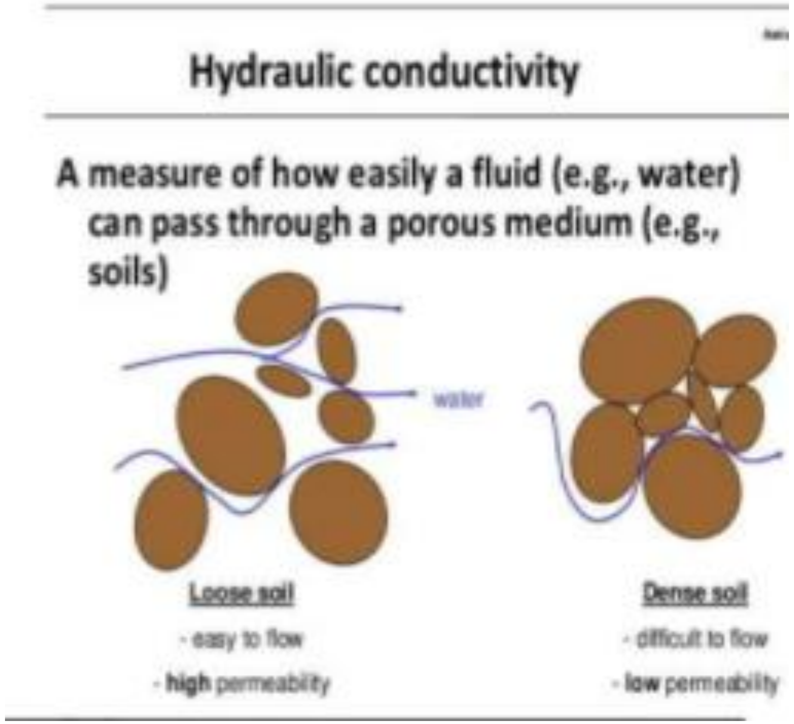
Dealing with Uncertainties: Unearthing Measures for Recovery
Organised by: PRIME MINISTER'S DEPARTMENT DEPARTMENT OF STATISTICS MALAYSIA, BANK NEGARA MALAYSIA, MALAYSIA INSTITUTE OF STATISTICS

INTRODUCTION

Introduction

- Uncertainty, due to lack of knowledge or natural variability. Depending on the location of the pumping well, the position of the observation well, the length of pumping, and the formation flow parameters, each measurement of drawdown from a pumping test is connected with a specific region of effect.

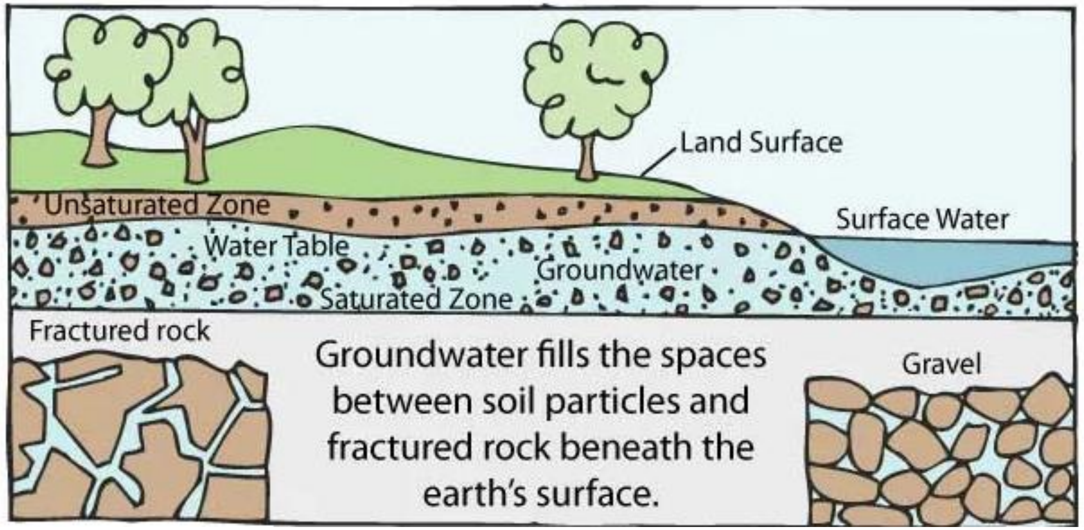
- Hydraulic conductivity is defined as the ease by which a fluid flows through a granular medium is denoted as (K) (Strobel, 2005). The estimated parameter K used among others are, to characterise the quantity of fluid injection possible in oil reservoirs, the movement of lixiviants in heap leaching mines, and the ease of groundwater flow in underground aquifers.



Introduction

- Groundwater is the world's most valuable resource. Nowadays, we have many kinds of studies using stochastic groundwater flow models. Because of that, Christakos et al. (1993), standard deterministic theories of flow and transport, demonstrated that when tested on natural phenomena, it is quite inadequate as they do not involve uncertainties in a parameter that can lead to enormous error.

- Therefore, stochastic approach is an alternative in groundwater behaviour study because it includes uncertainty in parameters for hydraulic conductivity groundwater flow. The stochastic model also provides theoretical and practical concepts to describe the mixtures of complexities and heterogeneities in this phenomenon.



Introduction

What is uncertainty in environmental study?

- Based on the literature study, the uncertainty associated with input factors such as hydraulic conductivity and porosity had not been considered in the models studied. As a result the groundwater flow model does not reflect the real condition of the groundwater that highly dependent on hydraulic conductivity that varies spatially.

- It is thus vital to include the uncertainty into the model parameters to improve the accuracy of the groundwater flow model prediction. In Malaysian alluvial and peat aquifers, a study requiring different hydrofacies and modified methods for estimating hydraulic conductivity must then be carried out.

Introduction

Research Objectives

to estimate the hydraulic conductivity

to predict the behaviour of the pumping rate

METHODOLOGY

Methodology

According to Simmons (2008), Darcy's Law holds because the medium's viscous resistance balances the driving forces of the fluid, gravity, and pressure. Darcy's law is expressed as:

$$Q = KA\left(\frac{dh}{dl}\right)$$

where:

Q = velocity of groundwater (m/d)

K = hydraulic conductivity (m/d)

A = cross-sectional area of flow (m^2)

$\frac{dh}{dl}$ = rate of hydraulic head per length.

Darcy's law can estimate the flow rate and water flow velocity within the aquifer. It can also help evaluate the average time taken of water flow from the aquifer's head to the point downstream.

Methodology

- The determination of aquifer parameters such as transmissivity, permeability, and storage coefficient is a critical part of groundwater resource investigation. Typically, transmissivity, permeability, and storage coefficient are determined using data from test-pump wells. The following equation was used to determine the transmissivity of the wells using the Theis recovery method:

$$T = \frac{Q}{4\pi \Delta s}$$

where:

T =transmissivity;

Q =pumping rate;

Δs = change in drawdown over t.

Methodology

- Theis (1935) developed the following approximate linear equation to predict drawdown in a homogeneous, isotropic, and nonleaky confined aquifer. With the assumption of a fully penetrating line sink that discharged at a constant rate prior to recovery:

$$s' = \frac{2.303Q}{4\pi T}[\log t - \log S]$$

where:

T = transmissivity;

Q = pumping rate;

s'= drawdown;

S = storativity during pumping;

t = time.

Methodology

Then, the hydraulic conductivity (K) is then estimated according to:

$$K = \frac{Q}{4\pi\Delta s b}$$

where:

b = aquifer thickness in meters,

Q = pumping rate; and

Δs = change in drawdown over t.

Methodology

- Natural problems are usually multi-dimensional and non-linear. Nonlinear regression is a form of regression analysis in which data is fit to a model and then expressed as a mathematical function. Simple linear regression relates two variables (X and Y) with a straight line (y = mx + b), while nonlinear regression relates the two variables in a nonlinear (curved) relationship.

A simple nonlinear regression model is expressed as follows:

$$\mathrm{Y} = \mathrm{f}(\mathrm{X}, \beta) + \epsilon$$

where:

X = vector of P predictors;

β = vector of k parameters;

F = (-) is the known regression function;

ϵ = the error term.

RESULT

Result

Data 1

Pumping Test Duration: 10.10 pm on 22/10/2009 to 10.10 am on 26/10/2009

Location : Jenderam Hilir, Selangor

Well No: DW1

Static Water Level=6.42m

Q=12.2m3/hr

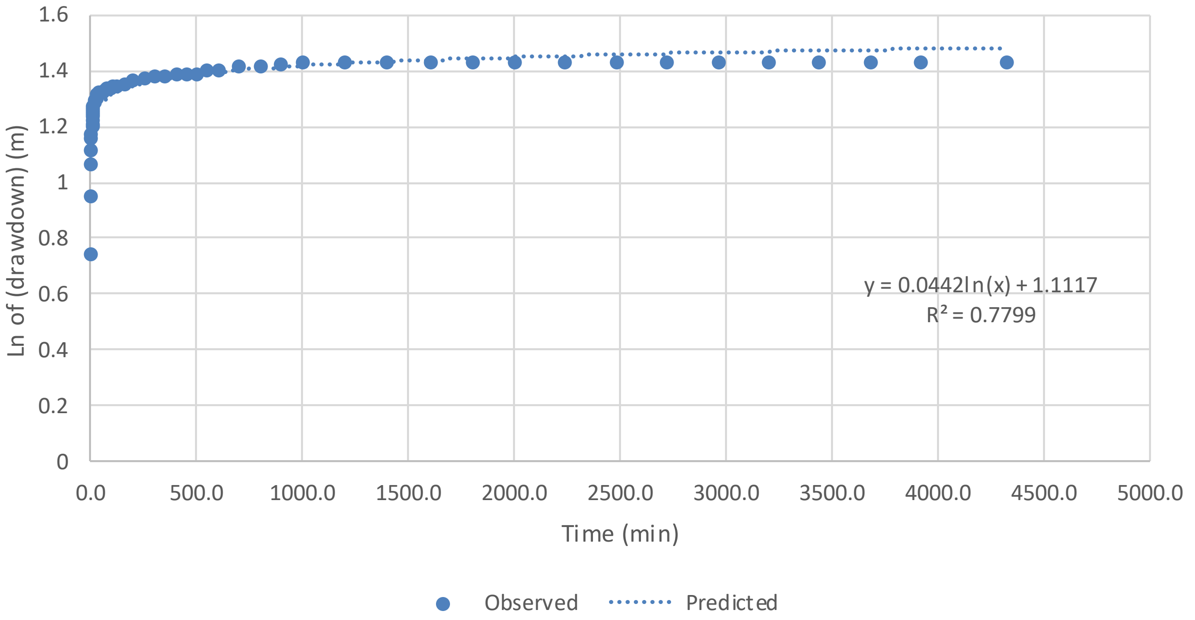
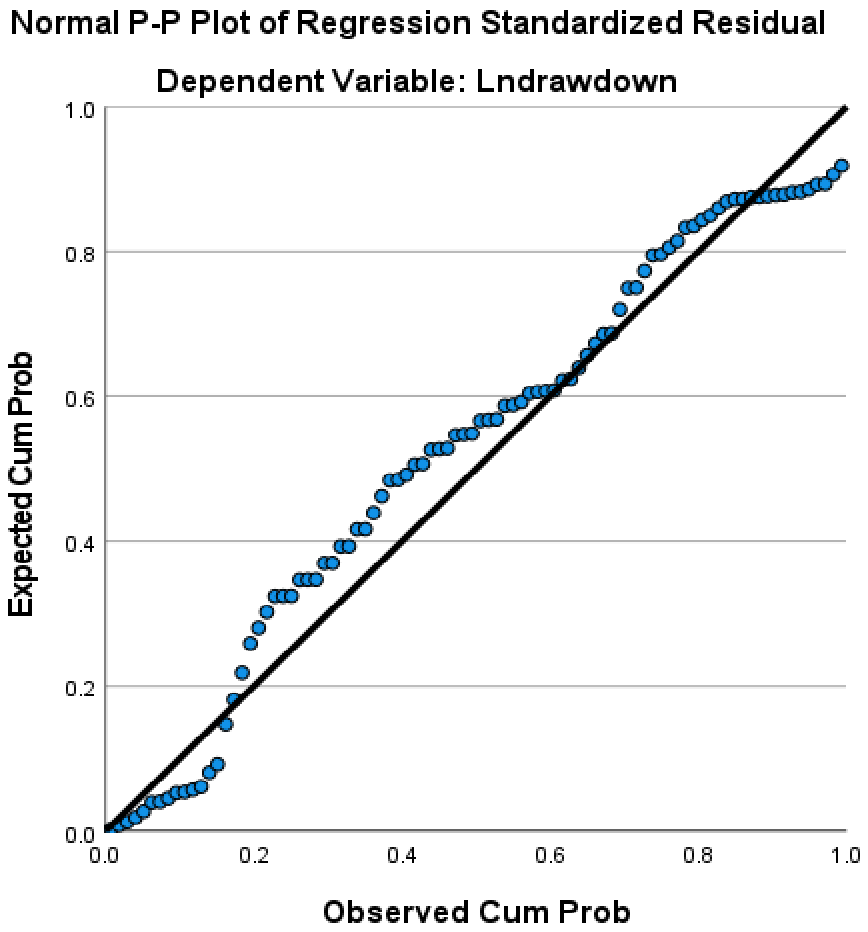


Figure 3.1 Graph log (drawdown) versus Time in minutes with predicted log regression line when Q=12.2m^3/hr.

Result

Normal P-P Plot Data 1



The normal probability plot of the residuals is approximately linear supporting the condition that the error terms are normally distributed when pumping rate, $Q=12.2m^3/hr$.

Result

Data 2

Pumping Test Duration: 10.00 pm on 05/10/2009 to 8.20 am on 7/12/2009

Location : Jenderam Hilir, Selangor

Well No: DW1

Static Water Level=5.19m

Q=51.6m^3/hr

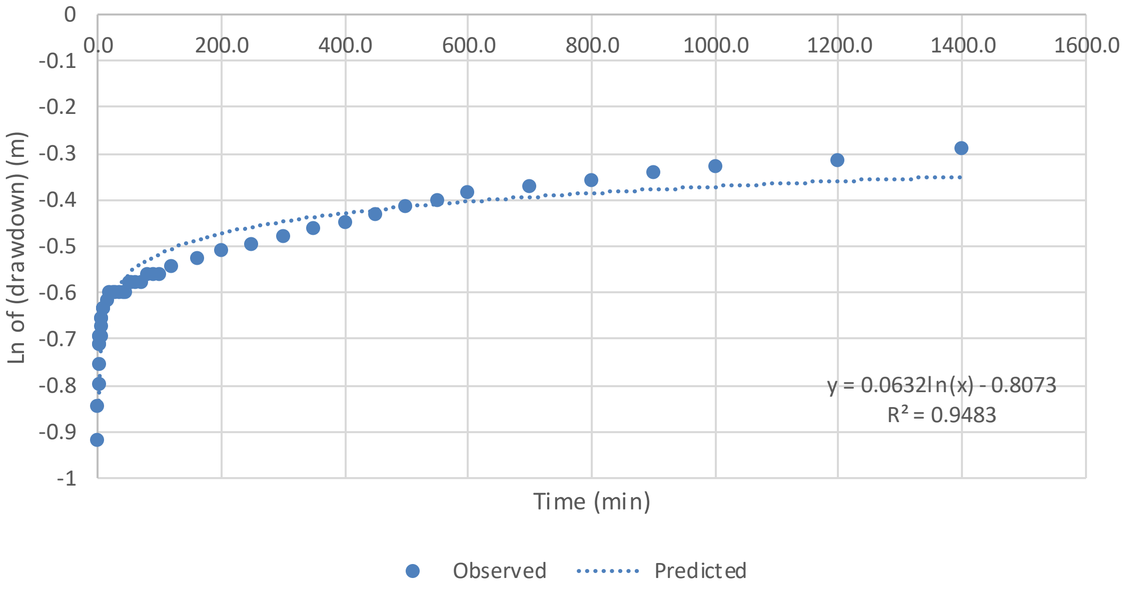
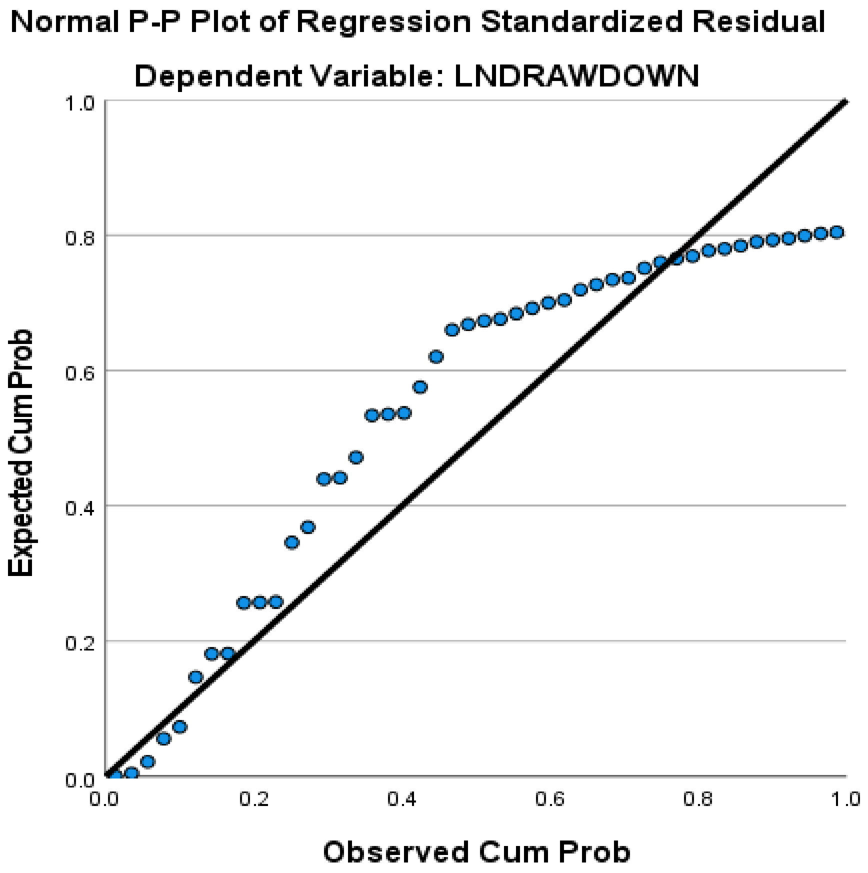


Figure 3.2 Graph log (drawdown) versus Time in minutes with predicted log regression line when Q=51.6m^3/hr.

Result

Normal P-P Plot Data 2



The normal probability plot of the residuals is approximately linear supporting the condition that the error terms are normally distributed when pumping rate, Q=51.6m^3/hr.

Result

Data 3

Pumping Test Duration: 1.45 pm on 22/04/2010 to 5.45 pm on 29/04/2010

Location : Jenderam Hilir, Selangor

Well No: DW2

Well Depth: 17.4m

Static Water Level=5.36

Q=90.2m^3/hr

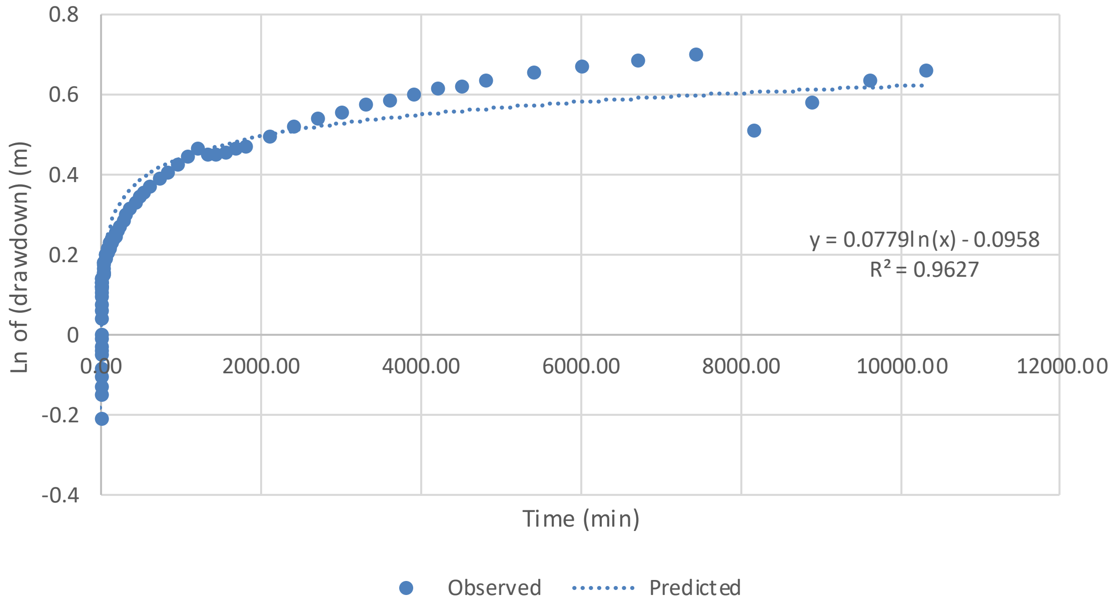
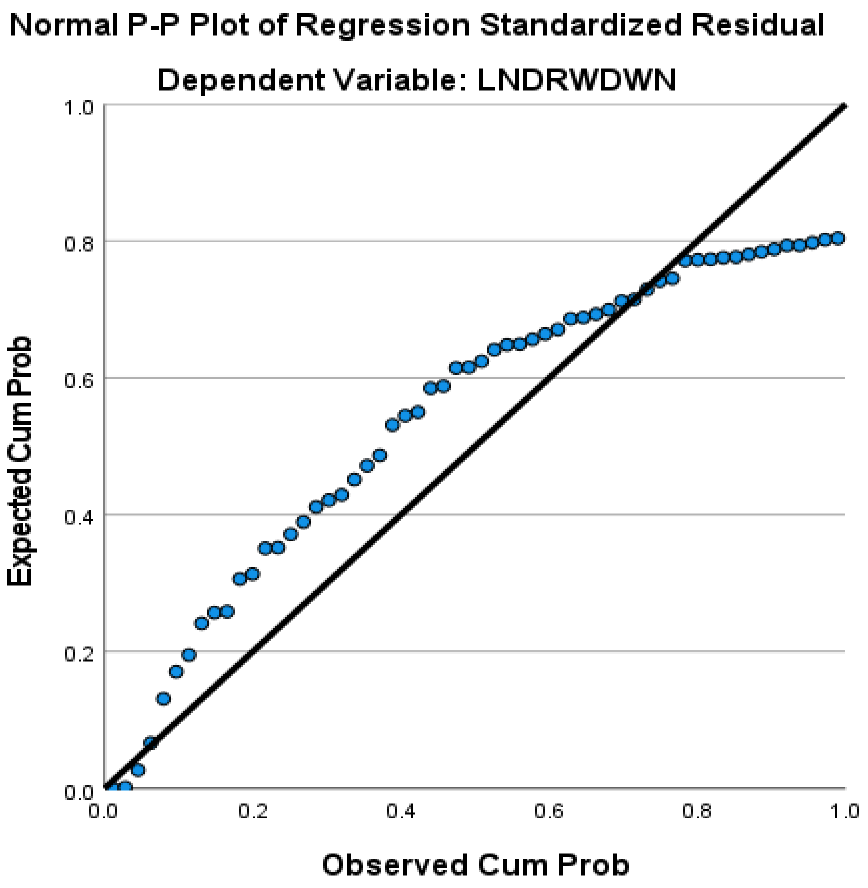


Figure 3.3 Graph log (drawdown) versus Time in minutes with predicted log regression line when Q=90.2m3/hr.

Result

Normal P-P Plot Data 3



The normal probability plot of the residuals is approximately linear supporting the condition that the error terms are normally distributed when pumping rate, $Q=90.2m^3/hr$.

Discussion and Conclusion

Discussion and Conclusion

- According to Figure 4.1, the pumping rate, Q=12.2m3/hr behave log normally. As a result, it is evident that we must use non-linear regression to predict groundwater hydraulic conductivity. From the Figure 4.2, the pumping rate, Q=51.6m3/hr behave log normally. Therefore, it is clear that we need to model the hydraulic conductivity of groundwater using non-linear regression. Figure 4.3 depicts how the pumping rate, Q=90.2m3/hr behave log normally.

- As a result, it is evident that we must use non-linear regression to predict groundwater hydraulic conductivity. For the parameter estimation, the hydraulic conductivity was calculated by Theis formula.

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